Signal Shaping Based Lattice Codes for MIMO Systems

Wei Wang

School of Electrical Engineering & Telecommunications
The University of New South Wales, Sydney, Australia

29 July 2016

Outline

Overview of MIMO Precoding Techniques

Precoding is a generalization of beamforming to support multi-stream transmission in multi-antenna wireless communications.

- Benefits of MIMO precoding
  - Multiplexing gain
  - Diversity gain
  - Antenna array gain
- Precoding is applied in 5G candidate technologies
  - Massive MIMO
  - Millimeter wave MIMO

Overview of MIMO Precoding Techniques

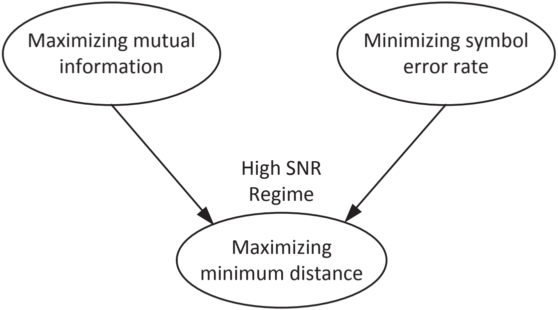
MIMO precoding techniques

- Point-to-point (single-user) MIMO precoding
  - Gaussian input – SVD and water-filling power allocation
  - Discrete input – lattice structured precoder
- Point-to-multipoint (multi-user) MIMO precoding
  - Linear precoding – MMSE/ZF/MRT precoding
  - Non-linear preocding – DPC precoding

[1] M. Vu and A. Paulraj, "MIMO wireless linear precoding," *IEEE Signal Process. Mag.,* vol. 24, no. 5, pp. 86-105, Sep. 2007.

[2] D. Kapetanovic, H. V. Cheng, W. H. Mow, and F. Rusek, "Lattice structures of precoders maximizing the minimum distance in linear channels," *IEEE Trans. Inf. Theory,* vol. 61, no. 2, pp. 908-916, Feb. 2015.

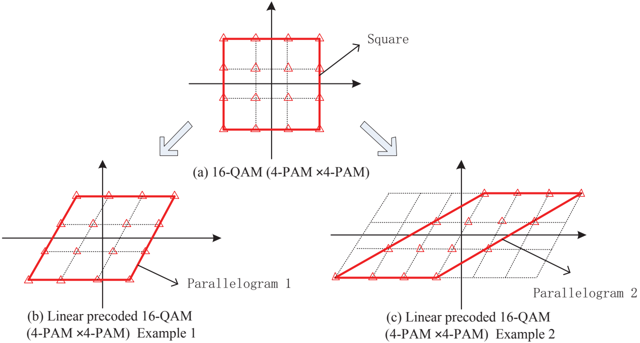
Precoding of MIMO



When SNR is large, both maximizing mutual information and minimizing symbol error rate is asymptotically equivalent to maximizing minimum codeword distance.

[3] M. Payaro and D. P. Palomar, “On the optimal precoding in linear vector Gaussian channels with arbitrary input distribution," in *Proc. IEEE International Symp. Inf. Theory (ISIT)*, Seoul, Korea, Jul. 2009, pp. 1085-1089.

MIMO Precoding – From A Lattice Perspective



- Precoded transmit signal $\mathbf{x} = \boldsymbol{\Sigma}^{-1}\mathbf{M}\mathbf{a}$, $\boldsymbol{\Sigma}^{-1}$ is channel inverse, $\mathbf{a}$ is modulation symbol vector, and precoder $\mathbf{M}$ is related to lattice structure as well as shaping region.
- Different generator matrices may have the same lattice structure but different shaping regions.

MIMO Precoding – State-of-the-Art

Facts of Precoder Design

- NP-hard problem [2][4];
- When L is relatively large, $\mathbf{M}$ has the perfect lattice structure [2];
- Optimal $\mathbf{M}$ is not always full-rank [4];
  - When cardinality L increases, the rank of optimal $\mathbf{M}$ increases;
  - When cardinality L decreases, the rank of optimal $\mathbf{M}$ decreases;
  - The rank of optimal $\mathbf{M}$ is related with the eigenvalues of channel matrix.

[2] D. Kapetanovic, H. V. Cheng, W. H. Mow, and F. Rusek, "Lattice structures of precoders maximizing the minimum distance in linear channels," *IEEE Trans. Inf. Theory,* vol. 61, no. 2, pp. 908-916, Feb. 2015.

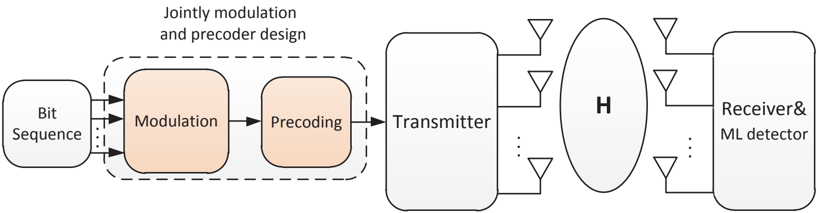
[4] D. Kapetanovic, F. Rusek, T. E. Abrudan, and V. Koivunen, "Construction of minimum Euclidean distance MIMO precoders and their lattice classifications," *IEEE Trans. Signal Process.,* vol. 60, no. 8, pp. 4470-4474, Aug. 2012.

Our Contributions

- We determine the optimal shaping region for MIMO;
- We design a low-complexity, yet efficient shaping-based codebook construction method;
- Our method is strictly better than present precoder-based codebook designs and is more robust under different MIMO channel conditions.

Outline

System Model



With singular value decomposition (SVD), the equivalent channel is $\mathbf{\Sigma} = \mathrm{diag}\{\lambda_1, \lambda_2, \cdots, \lambda_{2N_t}\}$, which means there are $2N_t$ parallel channels with decreasing channel gains.

Problem Description

- Design objective is to maximize

$$\mathrm{CFM}(C) \approx \underbrace{\frac{d^2_{min}(\Lambda)}{V(\Lambda)^{2/k}}}_{\gamma_c(\Lambda)} \cdot \underbrace{\frac{V(\Omega)^{2/k}}{P(\Omega)}}_{1/G(\Omega)} \cdot \underbrace{\frac{1}{L^{2/k}}}_{1/C_{Norm}} \quad (1)$$

  Constellation figure of merit $\mathrm{CFM}(C)$ is determined by three factors:

  (1) $\gamma_c(\Lambda)$ —- coding gain ;

  (2) $G(\Omega)$ —- normalized second moment of shaping region;

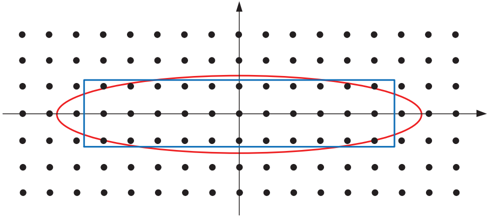
  (3) $C_{Norm} = L^{2/k}$ —- normalized cardinality.

- The key challenges of the codebook design are how to effectively shape the lattice in up to $2N_t$ spatial dimensions.

[5] J. H. Conway and N. J. A. Sloane, *Sphere Packings, Lattices and Groups.* 2nd ed. New York: Springer-Verlag, 1993.

Outline

Shaping Region



- Optimal hyperellipsoid shaping region

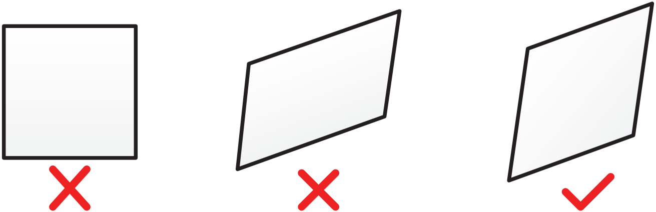
$$\Omega_E = \left\{ \mathbf{x} = (x_1, x_2, ..., x_{2N_t})^T | \mathbf{x}^\dagger \mathbf{\Sigma}^{-2} \mathbf{x} \leq r^2 \right\} \quad (2)$$

- Near-optimal hyperrectangle shaping region

$$\Omega_R = \left\{ \mathbf{x} = (x_1, ..., x_k)^T \mid x_i \in \alpha \cdot \left[-\frac{\lambda_i}{2}, \frac{\lambda_i}{2} \right), i = 1, ..., k \right\} \quad (3)$$

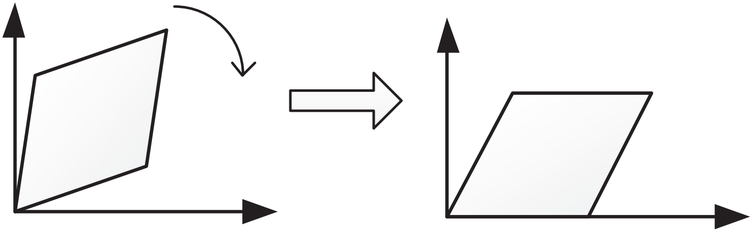
Shaping – How to Tile Your Floor

Shaping – Select Your Favorite Tiling Styles



Select lattice generator matrix $\tilde{\mathbf{M}}$, where $\tilde{\mathbf{M}}$ decides the type of tiling.

Shaping – Align the Tiles

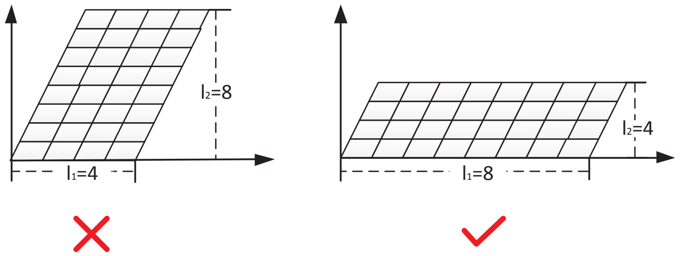


QR decomposition

$$\tilde{\mathbf{M}} = \mathbf{Q}\mathbf{M} \tag{4}$$

Rotate $\tilde{\mathbf{M}}$ and derive the upper triangular matrix $\mathbf{M}$.

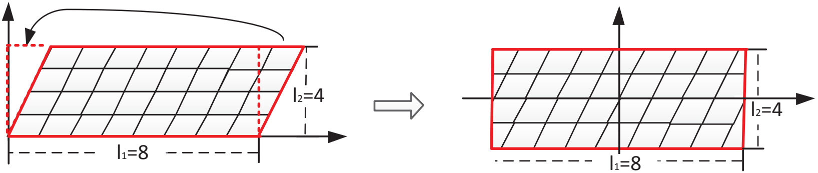
Shaping – Calculate Height and Width



The height and width of the parallelogram are decided by the orders of different PAM signals.

$$\tilde{\mathbf{x}} = \mathbf{M}\tilde{\mathbf{z}}$$
$$\tilde{z}_1 \in [0 \cdots l_1 - 1] \text{ and } \tilde{z}_2 \in [0 \cdots l_2 - 1] \quad (5)$$

Shaping – How to Get a Rectangle Using Parallelogram Tiles?



Hyperrectangle shaping: shift the outside points into rectangle.

$$z_i = \tilde{z}_i - l_i s_i \quad (6)$$

$$s_i = \left\lfloor \frac{1}{l_i} \left(\tilde{z}_i + \sum_{j=i+1}^{k} \frac{M_{i,j}}{M_{i,i}} z_j \right) \right\rceil \quad (7)$$

Codebook Construction

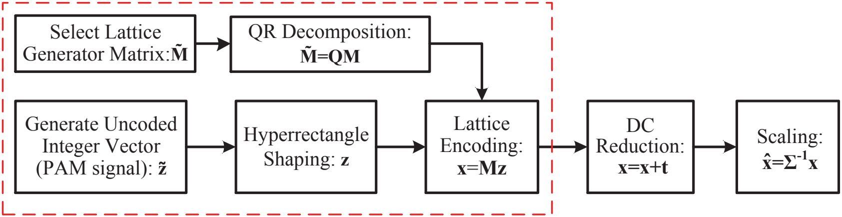


Figure: Flow chart of shaping-based codebook construction

Outline

2×2 MIMO

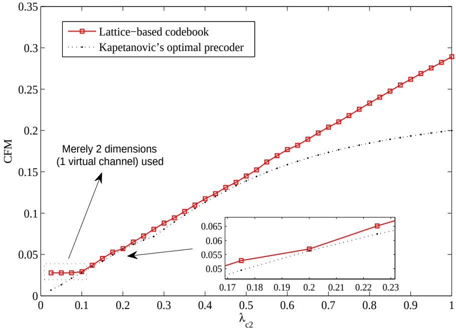


Figure: CFM comparison with optimal complex 2×2 precoder with 16-QAM in [5], and $\mathbf{\Sigma} = \text{diag}([1, \lambda_{c2}])$

Shaping-based codebook design is more robust under different channel conditions

[6] D. Kapetanovic, H. V. Cheng, W. H. Mow, and F. Rusek "Optimal two-dimensional lattices for precoding of linear channels," *IEEE Trans. Wireless Commun.* vol. 12, No. 5, May 2013.

3×3 MIMO

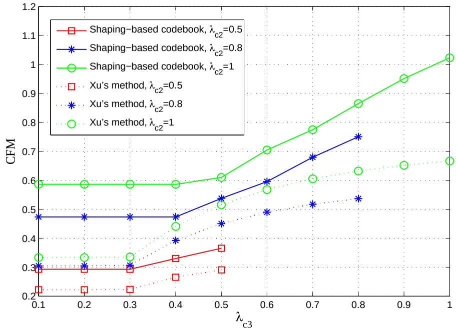


Figure: CFM comparison with suboptimal 3×3 precoder with 16-QAM in [6], and $\mathbf{\Sigma} = \mathrm{diag}([1, \lambda_{c2}, \lambda_{c3}])$

Our scheme is better because we use hyperrectangle shaping, while [7] does not consider shaping in their design.

[7] X. Xu and Z. Chen, "Recursive construction of minimum Euclidean distance-based precoder for arbitray-dimensional MIMO systems," *IEEE Trans. Commun.*, vol. 62, no. 4, pp. 1258-1271, Apr. 2014.

Outline

Conclusion

- A shaping-based codebook construction is proposed.
- Numerical results demonstrate that shaping-based codebook is better than other precoder-based codebook designs and is more robust under different MIMO channel conditions.

Questions

Thank you